Novel conditioning technique for systems subjected to constraints

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***Abstract*— A sliding mode reference conditioning for constrained linear systems is proposed which presents some distinctive features. In fact, contrarily to other control solutions, the sliding mode conditioning loop completely avoids saturation, so that the prescribed closed-loop behavior is never abandoned. As a consequence of this property as well as of the robustness features of sliding mode control, the closed-loop dynamics of the conditioning circuit and of the main loop are completely independent one of each other. So, no further analysis is necessary to guarantee stability of the controlled system during sliding mode conditioning.**

***Index Terms*— reference conditioning, constrained systems, sliding mode.**

I. INTRODUCTION

The interest in developing control strategies to reduce the undesirable effects of restrictions has led to an increasing number of publications addressing the problem from different viewpoints. For instance, using concepts of LQR (Turner and Walker, 2000), LPV (Wu and Grigoriadis, 1999), LMI's (Mulder *et al.*, 2001), SM (sliding mode) (Mantz and De Battista, 2002), robust control (Yoon *et al.*, 2002), etc. Many of these control solutions fit within the well-known 'two-step' design paradigm (Kothare *et al.*, 1994). In this methodology, the controller is firstly designed neglecting the restrictions (first step) and then a correction loop is incorporated which is active only when restrictions occur (second step). The aim of the auxiliary loop is to guarantee graceful degradation from the performance of the system without restrictions. Particularly, this work is exclusively concerned with the design of this correction loop (second step), whereas the linear controller design step is omitted.

Generally, limitations occur during transient responses to external excitations such as reference changes and disturbances. Then, many proposals can be interpreted as the conditioning of the reference to avoid problems caused by actuator saturation. For instance, one of the first and most cited contribution is the anti-windup (AW) conditioning technique developed for PID controllers (Hanus *et al.*, 1987). Although the conditioning is applied to the integral state of the controller, it is deduced from concepts of 'realizable reference' (Peng *et al.*, 1998) (This signal is interpreted as the reference that if it had been applied to the controller from the beginning, it would not have driven the actuator into saturation, i.e. the controller output would have coincided all the time with the input to the plant). A modified version of the conditioning technique is the so-called generalized conditioning technique proposed by Walgama and coworkers (1992). In this approach, a reference filter is incorporated, and conditioning is carried out on the filtered signal instead of directly on the reference. Helpfully, this generalized technique can be applied to a larger class of controllers. Among other contributions, Seron *et al.* (1995) and Hippe (2001) propose shaping the reference in a nonlinear fashion.

In this context of reference conditioning, a novel technique is introduced in this note. This new conditioning algorithm is developed within the framework of variable structure systems undergoing sliding regimes. This approach provides an extremely simple solution with very attractive features that differentiate it from other proposals. In fact, the proposed conditioning of the reference completely avoids the occurrence of restrictions, maintaining the main loop always closed. Then, this algorithm can be applied without distinction to both open-loop stable and unstable plants. Additionally, since the conditioning is carried out on the reference and the main loop is always closed, the main loop (linear) dynamics is affected neither by the restriction nor the conditioning loop. Moreover, due to the robustness properties of SM control, the conditioning loop dynamics is insensible to the process under control. Thus, the dynamics of the main control loop and of the SM conditioning loop are independent one of each other. This property has important implications. In fact, an additional study to investigate the stability of the controlled system during SM compensation is not necessary. Furthermore, the implementation and tuning of the SM algorithm is straightforward even in the case of a preexisting linear controller.

In contrast with other variable structure controllers, the aim of the control strategy proposed here is not to evolve in SM towards the equilibrium point. Contrarily, the sliding regime is intended as a transitional mode of operation. It is aimed at conditioning the rate of change of the reference in order to avoid that the controller output exceeds the limits of the actuator output or other variable of interest. Hence, once the system becomes able to evolve towards the linear operating region without reference conditioning, the SM compensating loop becomes inactive. Because of the nature of the proposed SM compensation, the main drawbacks of variable structure control (i.e. chattering and reaching time) can be ignored in the current application. In fact, the sliding regime is confined to the low-power side of the system, hence allowing the use of fast electronic devices. Besides, the conditioning loop is inactive until the system state reaches by itself the sliding surface, so there is no reaching mode.

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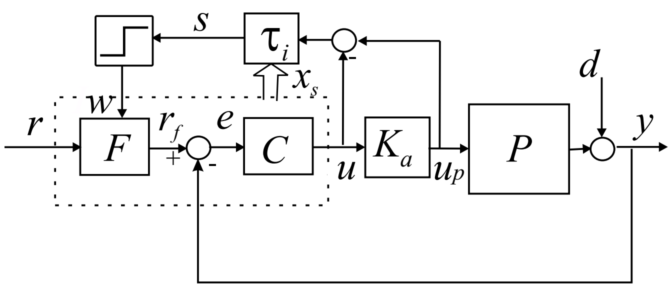

Fig. 1. Proposed reference conditioning via sliding mode.

II. AW REFERENCE CONDITIONING ALGORITHM VIA SM

Fig. 1 illustrates the control system with the proposed conditioning of the reference. Two loops can be distinguished: the main control loop and the SM reference conditioning loop. P represents the process under control which may be either open-loop stable or unstable. K_a is the actuator. C is a minimum phase rational controller of order m designed to accomplish the control specifications during linear operation. τ_i represents the set of SM feedback gains. F is a reference filter of first order similar to the one introduced by Walgama *et al*. (1992) It is worthy to mention that the filter has not of itself the aim of avoiding saturation after reference changes. In fact, this obvious solution would lead to an extremely conservative design. Contrarily, the compensation lies on the conditioning of the filter output as a function of the restrictions.

The actuator nonlinearity is characterized by

$$K_a : \begin{cases} u_p = \overline{u}_p & \text{if} \quad u > \overline{u}_p \\ u_p = u & \text{if} \quad \underline{u}_p \leq u \leq \overline{u}_p \\ u_p = \underline{u}_p & \text{if} \quad u < \underline{u}_p. \end{cases} \tag{1}$$

In this paper, saturation and rate limiting, which are typical actuator nonlinearities, are considered. In the former nonlinearity, the constant values $\overline{u}_p$ and $\underline{u}_p$ are the upper and lower limits of the actuator. In the latter case, $\overline{u}_p$ and $\underline{u}_p$ are linear increasing and decreasing functions of time.

Besides, the dynamic behavior of the process P, the controller C and the filter F are described by

$$P : \begin{cases} \dot{x}_p &= A_p x_p + b_p u_p \\ y &= c_p x_p, \end{cases} \tag{2}$$

$$F : \begin{cases} \dot{x}_f &= \lambda_f (x_f - r) + w \\ r_f &= x_f, \end{cases} \tag{3}$$

$$C : \begin{cases} \dot{x}_c &= A_c x_c + b_c e \\ u &= c_c x_c + d_c e. \end{cases} \tag{4}$$

Then, the open-loop dynamics of the conditioning loop is given by

$$\begin{aligned} \begin{bmatrix} \dot{x}_c \\ \dot{e} \end{bmatrix} &= \begin{bmatrix} A_c & b_c \\ 0 & \lambda_f \end{bmatrix} \begin{bmatrix} x_c \\ e \end{bmatrix} + \\ &+ \begin{bmatrix} 0 \\ \lambda_f (y - r) - \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w \qquad (5) \\ u &= c_c x_c + d_c e. \qquad (6) \end{aligned}$$

A. Conditioning algorithm for biproper controllers

For clarity of presentation, the conditioning algorithm is introduced first for biproper controllers C, i.e. for controllers having $d_c \neq 0$ (Note that PI controllers fall within this category). In the following subsection, the proposed algorithm is generalized to cope with a larger class of controllers.

In order to avoid windup by conditioning the filtered reference r_f, the following switching law is proposed:

$$\begin{cases} w = w^+ & \text{if} \quad s > 0 \\ w = 0 & \text{if} \quad s = 0 \\ w = w^- & \text{if} \quad s < 0 \end{cases} \tag{7}$$

where

$$s(u_p, x_s) = u_p - u, \tag{8}$$

being $x_s = \text{col}(x_c, e)$.

The objective of this discontinuous law is enforcing the following behavior (Utkin *et al*., 1999):

$$\begin{aligned} \lim_{s \to 0^+} \dot{s} < 0 \\ \lim_{s \to 0^-} \dot{s} > 0. \end{aligned} \tag{9}$$

In conventional VSS, this behavior leads to a sliding regime on the surface $s \equiv 0$. Conversely, in the current application this is not the case. Actually, the switching law (7) determines two sliding surfaces:

$$\begin{aligned} \overline{\mathcal{S}} = \{x_s | \overline{u}_p - u = 0\} \\ \underline{\mathcal{S}} = \{x_s | \underline{u}_p - u = 0\}. \end{aligned} \tag{10}$$

Whilst the actuator operates in its linear region, $s(u_p, x_s) = u_p - u = 0$ and the control signal is maintained at $w = 0$, i.e. no correction is made. Nevertheless, when u tries to exceed its upper bound ($s(u_p, x_s) < 0$), i.e tries to cross $\overline{\mathcal{S}}$, the control signal w switches to w^-. Similarly, when u intends to fall below its lower bound ($s(u_p, x_s) > 0$) and cross $\underline{\mathcal{S}}$, w switches to w^+. Then sliding regimes are feasible on the surfaces $\overline{\mathcal{S}}$ and $\underline{\mathcal{S}}$.

A necessary condition to accomplish (9) is that $s(u_p, x_s)$ must be of relative degree one with respect to w (Sira-Ramírez, 1989). Note that this is true for all biproper controllers. In fact, the error e appears implicitly in (8) through the forward gain d_c. Then, properly choosing the switching levels $w^{\pm}$, sliding regimes are established once the sliding surfaces are reached (i.e. once the controller output reaches one of its bounds). A sliding regime on $\overline{\mathcal{S}}$ or $\underline{\mathcal{S}}$ implies that the filtered reference r_f is continuously adjusted so that the controller output never goes outside the linear region of the actuator. Thus, windup and other related problems are completely avoided.

During a sliding regime, x_s is constrained to the sliding surface, thus existing a linear dependence among the state coordinates. In fact, from (8) and (6), the error e can be expressed as a function of x_c: $e = d_c^{-1}(\hat{u}_p - c_c x_c)$ where $\hat{u}_p$ is used to represent either of the saturation limits $\overline{u}_p$ or $\underline{u}_p$. This dependence can be used to obtain the closed-loop sliding dynamics (Hung *et al*., 1993). Effectively, replacing this equation in (5), the reduced-order sliding dynamics yields:

$$\dot{x}_c = Q_c x_c - b_c d_c^{-1} \hat{u}_p \tag{11}$$

$$Q_c = (A_c - b_c d_c^{-1} c_c). \tag{12}$$

As it can be easily checked, the eigenvalues of Q_c are the zeros of the transfer function of C. Since C is a minimum phase controller, the SM conditioning loop is stable. This dynamics is not seen from the controller output which is fixed at $\hat{u}_p$ during the sliding regime. Moreover, this (inverse) dynamics presents interesting robustness properties (note that it depends only on controller parameters) that will be formalized immediately after the generalization of the method.

B. Generalization to strictly proper controllers

In this subsection, the algorithm previously introduced is extended to handle minimum phase strictly proper controllers.

The dynamics of the conditioning loop is still represented by (5)–(6) where now $d_c = 0$. Let ρ be the relative degree of the controller C with respect to its input e. It is convenient to transform the open-loop conditioning dynamics (5)–(6) into the normal canonical form:

$$\left\{ \begin{array}{rcl} \dot{u}_1 & = & u_2 \\ \dot{u}_2 & = & u_3 \\ \cdots & = & \cdots \\ \dot{u}_{\rho-1} & = & u_\rho \\ \dot{u}_\rho & = & a_\mu \mu_c + a_\eta \eta_c + b\, e \\ \dot{\eta_c} & = & P\mu_c + Q\eta_c \\ \dot{e} & = & \lambda_f e + \lambda_f (y - r) - \dot{y} + w \\ u = u_1 & & \end{array} \right. \tag{13}$$

where $\mu_c = [\ u_1 \quad u_2 \quad \cdots \quad u_\rho\ ]^\top$ comprises the controller output and its first $(\rho - 1)$ derivatives, η_c is a set of $(m - \rho)$ linearly independent state coordinates, and $b \neq 0$. In this form, the zeros of the transfer function of C are the eigenvalues of Q.

Unfortunately, for strictly proper controllers ($\rho > 0$), (8) does not verify the reaching condition (9) to establish a sliding regime on the associated sliding surfaces (Sira-Ramírez, 1989). To satisfy this condition, the switching law is reformulated as follows:

$$s = u_p - u - \sum_1^\rho \tau_{i+1} u^{(i)} \tag{14}$$

where $u^{(i)}$ is the i^{th} derivative of the controller output u. This new switching function has relative degree one with respect to the control signal w. Actually, it can be rewritten in terms of the new coordinates $\tilde{x}_s = \mathrm{col}(\mu_c, \eta_c, e)$:

$$s(u_p, \tilde{x}_s) = u_p - \sum_1^\rho \tau_i u_i - \tau_{\rho+1}(a_\mu \mu_c + a_\eta \eta_c + b\, e) \tag{15}$$

where $\tau_1 = 1$, and differentiating with respect to time once, it appears the control signal w. This switching law determines two sliding surfaces

$$\begin{array}{l} \overline{\mathcal{S}} = \{\tilde{x}_s | s(\overline{u}_p, \tilde{x}_s) = 0\} \\ \underline{\mathcal{S}} = \{\tilde{x}_s | s(\underline{u}_p, \tilde{x}_s) = 0\}. \end{array} \tag{16}$$

associated to the upper and lower limits of the actuator, respectively. With $\hat{\mathcal{S}} = \{\tilde{x}_s | s(\hat{u}_p, \tilde{x}_s) = 0\}$, we will refer without distinction to any of these surfaces. Once any of these sliding manifolds is reached, a sliding regime will be established on it provided the signal levels $w^\pm$ are selected appropriately.

Again, during a sliding regime on $\hat{\mathcal{S}}$, there exists a linear dependence among the state coordinates. In fact, $s(\hat{u}_p, \tilde{x}_s) = 0$ can be solved for e and replaced in (13). Thus, the reduced-order sliding dynamics yields (Hung *et al.*, 1993)

$$\left\{ \begin{array}{rcl} \dot{u}_1 & = & u_2 \\ \dot{u}_2 & = & u_3 \\ \cdots & = & \cdots \\ \dot{u}_{\rho-1} & = & u_\rho \\ \dot{u}_\rho & = & \tau_{\rho+1}^{-1} \left(\hat{u}_p - \sum_1^\rho \tau_i u_i\right) \\ \dot{\eta_c} & = & P\mu_c + Q\eta_c. \end{array} \right. \tag{17}$$

Clearly, during SM operation, the dynamics of the controller output is governed by the first ρ files of (17). Consequently, if the coefficients τ_i are chosen so that the roots of the polynomial $\sum_0^\rho (\tau_{i+1} s^i)$ are all real and negative, then the sliding regimes on $\hat{\mathcal{S}}$ will be stable and u will not present overshoots. That is, once a sliding regime is established, the controller output will tend exponentially towards its saturation limit until the system reenters linear operation. Consequently, the actuator never saturates. On the other hand, the hidden dynamics of the controller will be governed by the eigenvalues of Q, i.e. by the zeros of the transfer function of C. Since C is of minimum phase, this hidden dynamics is also stable.

Remark 1: Because of the robustness properties of sliding modes, the dynamics of the conditioning loop is independent of the main loop dynamics. Effectively, the second and third terms of the right hand side of (5) can be interpreted as the disturbance and controlled vector fields of the conditioning loop, respectively. Clearly, the disturbance satisfies the matching condition (Sira Ramírez, 1988), i.e. $\mathrm{col}(0, \lambda_f(y-r) - \dot{y}) \in span\,(\mathrm{col}(0, w))$ Then, as (17) and (11) corroborate, the SM conditioning dynamics is insensible to the process output y and its derivative $\dot{y}$. Moreover, the dynamics of the controller output is completely governed by the feedback gains τ_i. Effectively, neither the process P nor the main loop controller C affects the dynamics of the control signal u while the SM conditioning loop is active.

Remark 2: The dynamics of the main loop, with input r_f and output y, is also independent of the conditioning loop (being active or inactive). In fact, the main loop dynamics remains governed by the controller C. Two facts make this possible: (1) The conditioning circuit acts on the filtered reference r_f, taking no action within the main loop; (2) The conditioning circuit ensures that no limitation occurs, guaranteing closed-loop linear operation of the main loop. Thus, the dynamics of the main loop is always described by

$$\begin{array}{rcl} \left[\begin{array}{c} \dot{x}_p \\ \dot{x}_c \end{array} \right] & = & \left[\begin{array}{cc} A_p - b_p d_c c_p & b_p c_c \\ -b_c c_p & A_c \end{array} \right] \left[\begin{array}{c} x_p \\ x_c \end{array} \right] + \\ & + & \left[\begin{array}{c} b_p d_c \\ b_c \end{array} \right] r_f \\ y & = & [\ c_p \quad 0\ ] \left[\begin{array}{c} x_p \\ x_c \end{array} \right]. \end{array} \tag{18}$$

Then, the eigenvalues of the main loop dynamics are completely independent of the SM conditioning loop.

The independence of both loops assures that the activation of the conditioning system affects neither the stability nor the dynamics of the main loop. Consequently, contrary to what happens with previous control solutions found in literature, no further analysis is necessary to guarantee the stability of the system in the presence of the compensating loop. Moreover, since the main loop is always closed even in the presence of constraints, i.e. $u = u_p$, the conditioning technique can also be applied to unstable plants.

III. Illustrative examples

A. Example 1

Consider an unstable plant together with a PI controller. The transfer functions of the plant and controller are

$$P(s) = \frac{1}{s-10} \tag{19}$$

$$C(s) = 50\frac{s+10}{s}. \tag{20}$$

Note: The design of C corresponds to the first step of the control system design, and falls beyond the scope of this work. Here, both the plant and the linear controller are given and an AW loop is designed for them to degrade gracefully from the linear response.

The performance of the closed-loop systems in the absence of actuator constraints is observed in Fig. 2. The response of the controlled variable y (2a) showing a 25% overshoot, is assumed to be the desired response of the system. Fig. 2b depicts the control signal.

Fig. 3 shows the simulation results for the case of a bounded actuator. In this case, the actuator output is limited to $u_p \in [-15, 15]$ whereas its time derivative is constrained to $\dot{u}_p \in [-600, 600]$. It can be observed in Fig. 3b that rate limiting and saturation occur until the actuator enters its linear region at t_L ($u_p = u$). As a result, a large overshoot and a long settling time appears on the controlled variable y, revealing a typical windup behavior (solid line of Fig. 3a).

To avoid windup, the proposed SM reference conditioning is employed. For this purpose a first order filter is incorporated. Its pole is chosen much faster than the process dynamics ($\lambda_f = -50$) so that it does not appreciably affect by itself the linear response. The auxiliary control signal w switches according to the switching law $s(u_p, x_s) = u_p - u$. The claimed behavior of the SM conditioning loop is corroborated in Fig. 4. The solid line of Fig. 4a depicts the process output y whereas Fig. 4b displays the controller output u when the SM conditioning is employed. For comparative purposes, the linear response (i.e. without saturation) is repeated in dashed line. Fig. 4b corroborates that the controller output never falls into restrictions (in fact, u coincides with u_p all the time), thus completely eliminating windup. Clearly, a sliding regime is established immediately after the reference step avoiding rate limiting. From t_1, the sliding mode is devoted to avoid actuator saturation. Finally, at t_2 the SM correction becomes inactive. The conditioned reference r_f, i.e. the reference signal that completely avoids constraints, is also displayed in Fig. 4a (dotted line). This figure shows that the large overshoot is effectively corrected, satisfying the aim of the AW loop of degrading as gracefully as possible from the linear response.

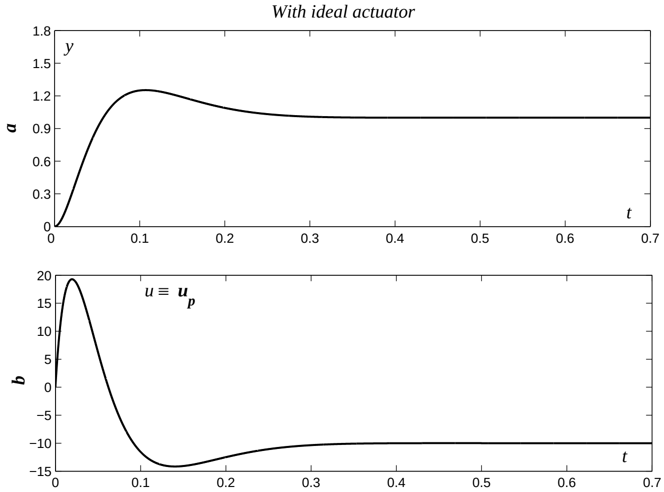

Fig. 2. (a) Controlled variable y, and (b) control signal u of the system without constraints.

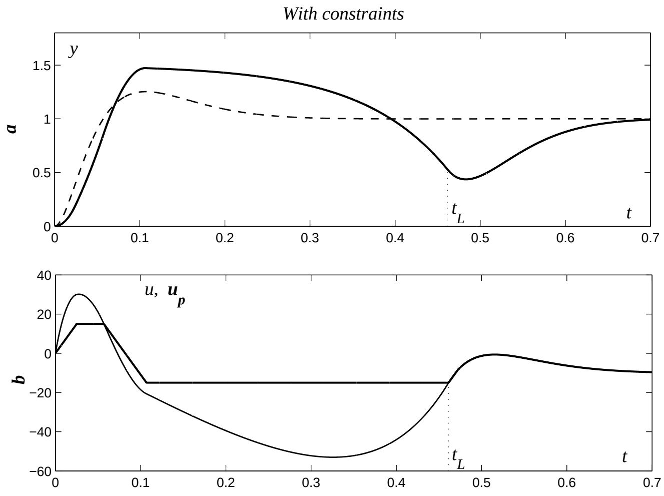

Fig. 3. (a) Controlled variable y, and (b) control and actuator outputs u and u_p of the constrained system.

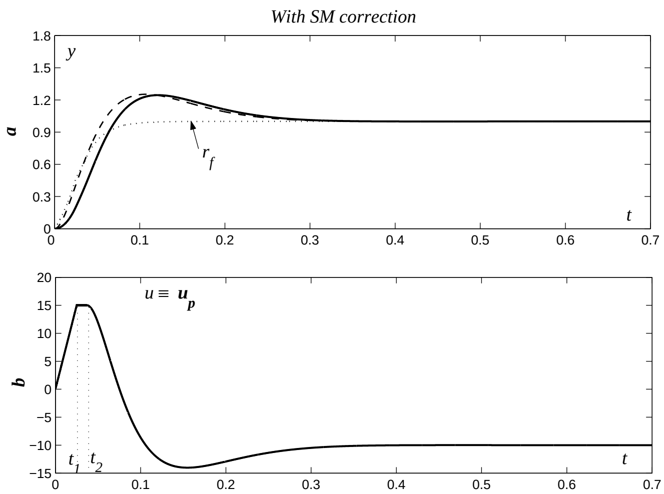

Fig. 4. a) Controlled variable y and filtered reference (dotted), and (b) control and actuator outputs $u \equiv u_p$ of the constrained system with SM conditioning.

Remark 3: No control effort is done to enforce the state to reach the sliding surface. Moreover, evolution within the linear region is the desired mode of operation. So, no attention has to be paid to the reaching mode.

Remark 4: As a result of the SM anti-windup conditioning, the controller and the actuator outputs coincide at every time. Since no saturation occurs, the main control loop remains closed all the time and its dynamics is not altered by the conditioning loop. So, no further stability analysis of the main loop is necessary.

Remark 5: The SM conditioning can be applied to open-loop unstable plants. It can be seen that, in order to force operation in the linear region, the process instability is transferred to the reference. This fact, in other context, has been already stressed in Seron *et al*. (1995). Effectively, during the sliding regime on $\overline{\mathcal{S}}$, r_f increases exponentially. Obviously, this mode of operation finishes when the system reenters the linear region. Effectively, if the closed-loop dynamics of the main loop is faster than the one corresponding to the unstable poles of P, the error e will diminish rapidly. Then, when r_f approaches r, the main loop will be able to operate in the linear region, deactivating the SM compensation. From then, r_f converges to r with the time constant of the filter F.

B. Example 2

Consider now the following example. The equation

$$\dot{x} = \begin{bmatrix} 0 & .01 & 0 \\ -50 & -1 & 1 \\ -.001 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 50 \\ .001 \end{bmatrix} r \qquad (21)$$

corresponds to a closed-loop system. The first two files describe the dynamics of the process whereas the last is the integral state of the PI controller. It is supposed here that, for safety operation of the system, the variable x_2 must be upper-bounded to $x_{2lim} = 20$. Naturally, it is assumed that x_2 is accessible.

To fit within the general framework previously developed, the controller and process should be redefined. Effectively, equation (21) can be reinterpreted as a process described by the first file and a strictly proper controller described by the last two files.

In order to meet with this constraint on x_2 (which is here seen as a control signal), an SM conditioning of the filtered reference is proposed. Unfortunately, the obvious switching law $s(x) = x_{2lim} - x_2$ does not have relative degree one with respect to w. So, a sliding regime cannot be established on it and x_2 might cross the surface $s(x) = 0$ and evolve towards dangerous operating regions. To overcome this obstacle, a new switching law defined by $s(x) = x_{2_{lim}} - x_2 - \tau\dot{x}_2 = 0$ is proposed. Note that error feedback is implicit in $\dot{x}_2 = -x_2 + 50e + x_3$. To ensure safety operation, the SM reference conditioning begins even before x_2 reaches its upper-bound. During the sliding regime, x_2 converges exponentially towards its limit value with time constant τ. Fig. 5a shows the smooth convergence of x_2 to its bound at $x_{2_{lim}} = 20$ with the fast sliding dynamics. Fig. 6 displays the associated system trajectory in the phase plane $(x_2, \dot{x}_2)$. The SM protection acts along the line AB. Obviously, the slope of the sliding surface as well as the sliding dynamics is given by the sliding gain τ. Once the system trajectory points towards the safety region from both sides of the sliding surface, the system naturally evolves in this region without reference conditioning towards its equilibrium point. Finally, Fig. 5b shows the graceful degradation of the controlled variable x_1 caused by the SM conditioning.

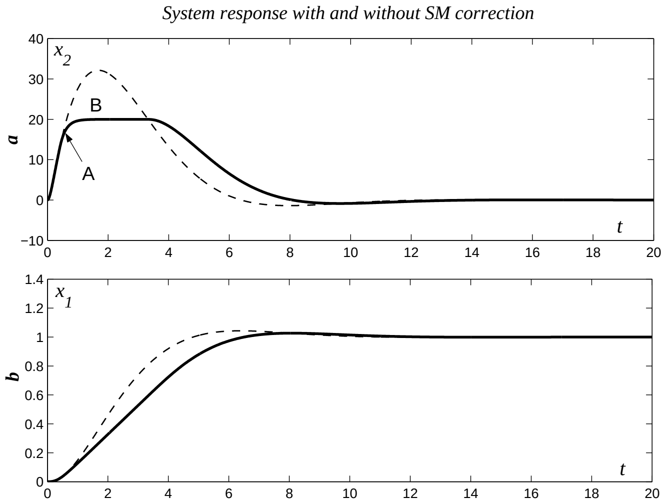

Fig. 5. System response with and without SM conditioning, a) variable x_2, b) variable x_1.

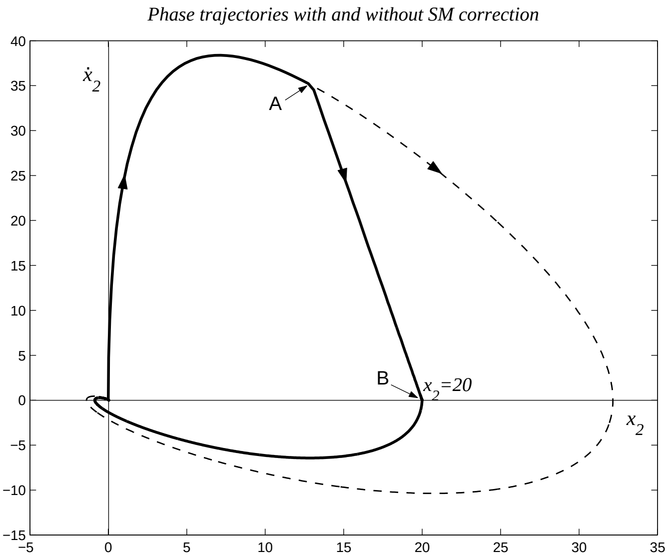

Fig. 6. Phase trajectories in $(x_2, \dot{x}_2)$ of the system response with and without SM conditioning.

Remark 6: As this example shows, the proposed SM algorithm originally developed to cope with actuator nonlinearities can be extended to overcome problems associated to other kind of restrictions.

IV. CONCLUSIONS

An anti-windup algorithm based on the reference conditioning concept is developed using tools of sliding mode control. The most distinctive feature of the proposed methodology is the independence of the main control loop and SM conditioning loop dynamics. Effectively, since the conditioning loop acts on the reference and nonlinearities are completely avoided (in contrast with other approaches that are intended to minimize the degradation in the presence of the problem), the closed-loop linear behaviour of the main loop is never abandoned. Consequently, no further analysis is necessary to guarantee the stability of the main loop. In addition, due to the robustness properties of SM control, the conditioning loop dynamics is insensible to the process output evolution. Moreover, the controller output dynamics is completely governed by the gains of the designer-chosen switching function. Finally, the implementation and tuning of the SM conditioning algorithm is extremely simple.

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